AP Physics – Satellite Stuff

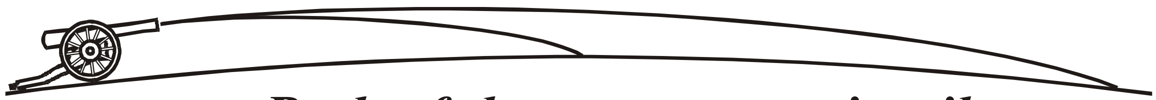
One of Sir Isaac Newton's greatest claims to fame is his explanation of how the planets orbit the sun. That and the ability to compute the orbits, orbital speeds, orbital periods, &tc. Before Newton, the motion of the heavens was a mystery. After Newton, the motion of the heavens was an explainable physical phenomenon.

Let's do us a "mind experiment". This is an experiment where you think instead of do. Anyway, picture a cannon that is set to fire horizontally. What does the path of the projectile look like?

Path of short range projectile

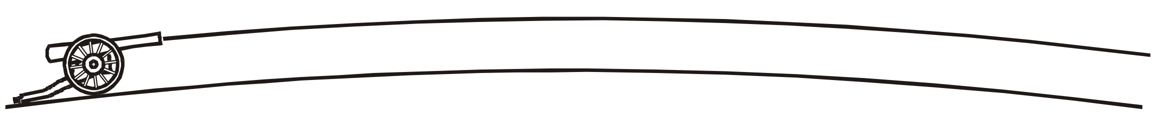
The projectile will follow a curved path. This is because it is being accelerated downwards by the force of gravity. The greater the velocity of the projectile, the farther it will go before it strikes the Earth.

The Earth, however, is not flat, although over short distances we can pretend that it is. So what actually is happening is that the projectile moves over and falls to the ground on a curved surface. So we have a curved path and a curved surface. We have to take the curvature of the Earth into account when firing long range projectiles. Possible paths would look like this:



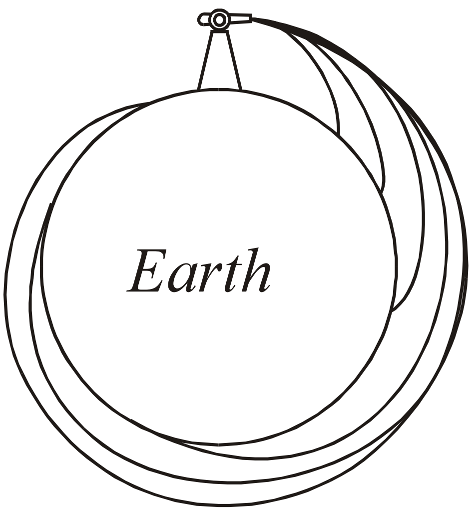
Path of long range projectile

Again, the greater the velocity of the projectile, the greater the range. Newton showed that if the velocity was great enough, the curving path of the falling projectile would match the curved surface of the earth and the falling projectile would never actually hit the Earth. Here is a drawing of this.

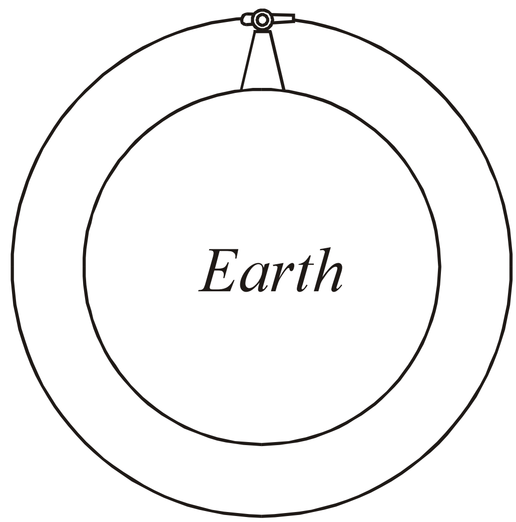


Path of projectile with same curvature as surface of the earth

Newton imagined a mountain on the earth that was so high that its summit was outside of the earth's atmosphere (this eliminates friction with air). On top of the mountain is a powerful cannon. The cannon fires a projectile horizontally. The projectile follows a curved path and eventually hits the earth. Now we add more gunpowder to the charge and fire another cannonball. This cannonball will travel a greater distance before it too hit the surface of the earth. We keep firing the gun with a bigger and bigger charge. The cannonball goes further and further before it strikes the earth. Eventually the velocity is great enough so that the curved path of the projectile matches the curved surface of the earth and the cannonball never gets closer to the planet's surface. It keeps falling forever.



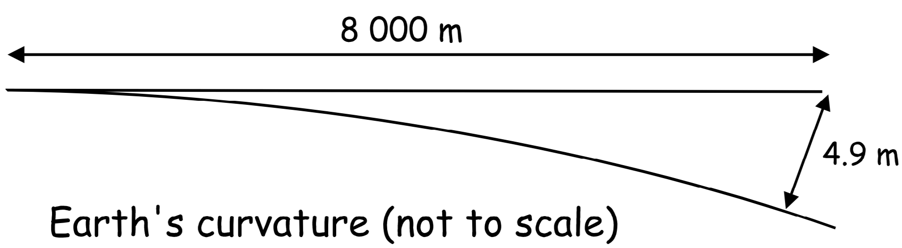
Path of projectile fired with larger and larger charges



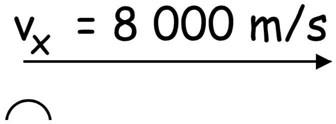
With just the right velocity, the projectile never reaches the earth's surface

That basically is your orbit.

The earth is not flat. It is a sphere and its surface has a fairly constant curvature. The surface drops 4.9 m in 8 000 m of horizontal travel.



If we launch a cannonball with a velocity of 8 000 m/s, it will fall a distance of 4.9 m and travel a horizontal distance of 8 000 m in one second. This means that it will stay at the same height above the earth's surface throughout its path. Of course, if we did this near the surface, we'd have the air slowing the projectile down. We'd also have to worry about the cannonball running into houses and mountains and trees and so forth. Above the atmosphere, however, all these impediments are eliminated.



The orbit of the everyday celestial object is described by a combination of the law of gravity, Newton's three laws, and the stuff we just learned about, circular motion.

Orbital Equations: Let us assume that the orbit of a satellite about the earth (or any other massive body) is a circle. Most orbits, the Physics Kahuna must point out, are not circles but are instead ellipses. This was discovered by Johanes Keppler in the 1600's. But let's keep it simple and look at a circular orbit.

In order to have a circular path, a centripetal force is required. This is supplied by the force of gravity between the two bodies.

So we can set the centripetal force equal to Newton's law of gravity:

$$F = G\frac{m_1 m_2}{r^2} \qquad \textbf{\textit{gravity}} \qquad F_C = \frac{m_2 v^2}{r} \qquad \textbf{\textit{centripetal force}}$$

Set them equal to one another: $$G\frac{m_1 \cancel{m_2}}{r^{\cancel{2}}} = \frac{\cancel{m_2} v^2}{\cancel{r}}$$

Notice how the mass of the object canceled out.

$$v^2 = \frac{Gm_1}{r} \qquad v = \sqrt{\frac{Gm_1}{r}}$$

This gives us an equation for the orbital velocity:

$$v = \sqrt{\frac{Gm}{r}}$$

The mass, ***m***, in the equation is the mass of the body being orbited. If we are talking about a planet orbiting the sun, then the mass we would use would be that of the sun. The mass of the satellite cancels out, so its mass is not involved in the orbital velocity equation at all.

The equation for the orbital velocity will not be given you on the AP Physics Test. So be prepared to derive it if you need it.

Period of satellite: This is another simple derivation job. The period of a satellite is ***T***, the time to make one orbit. What would be the period of the earth around the sun?

Let's develop the equation for the period of a satellite. We'll use the equation for distance and solve it for the time:

$$v = \frac{x}{t} \qquad t = \frac{x}{v}$$

d, the distance traveled is the circumference of the orbit. We know that it would be:

$$x = 2\pi r$$

So we can plug that in to the equation we solved for time:

$$t = \frac{2\pi r}{v}$$

but v is also given by the equation we just derived for the orbital velocity:

$$v = \sqrt{\frac{Gm}{r}}$$

If we plug the orbital velocity into our working equation, i.e., put them together, we get:

$$t = \frac{2\pi r}{\sqrt{\frac{Gm}{r}}}$$

Square both sides: $$t^2 = \frac{4\pi^2 r^2}{\frac{Gm}{r}}$$

Clean up everything up nice and neatlike using our potent algebra skills:

$$t^2 = 4\pi^2 r^2 \frac{r}{Gm} \qquad t^2 = \frac{4\pi^2 r^3}{Gm}$$

$$t = \sqrt{\frac{4\pi^2 r^3}{Gm}} \qquad t = 2\pi\sqrt{\frac{r^3}{Gm}}$$

And we end up with an equation for the period of a satellite. Again the mass in the thing is the mass of the body being orbited:

$$t = 2\pi\sqrt{\frac{r^3}{Gm}}$$

Now let's do some exciting problems.

- What is orbital velocity of the earth around the sun? The sun has a mass of 1.99 x 10^{30}kg, the mean distance from the earth to the sun is 1.50 x 10^{11} m.

$$v = \sqrt{\frac{Gm}{r}} = \sqrt{6.67\,x\,10^{-11}\,\frac{\cancel{kg}\cdot\cancel{m}\cdot m^2}{s^2\cdot\cancel{kg}^2}\left(1.99\,x\,10^{30}\,\cancel{kg}\right)\frac{1}{1.50\,x\,10^{11}\,\cancel{m}}}$$

$$v = \sqrt{8.85\,x\,10^{8}\,\frac{m^2}{s^2}} = \boxed{2.97\,x\,10^{4}\,\frac{m}{s}}$$

- A satellite is in a low earth orbit, some 250 km above the earth's surface. r_{earth} is 6.37 x 10^6 m and m_{earth} = 5.98 x 10^{24} kg. Find the period of the satellite in minutes.

r is the radius of the earth plus ***y***, the height of the satellite. $r = r_{earth} + y$

$$y = 250\,\cancel{km}\left(\frac{10^3\,m}{1\,\cancel{km}}\right) = 0.25\,x\,10^6\,m$$

$$r = 6.37 \text{ x } 10^6 \text{ m} + 0.25 \text{ x } 10^6\text{m} = 6.62 \text{ x } 10^6 \text{ m}$$

$$T = 2\pi\sqrt{\frac{r^3}{Gm}} = 2\pi\sqrt{\frac{\left(6.62\,x\,10^6\,\cancel{m}\right)^3}{6.67\,x\,10^{-11}\,\frac{\cancel{kg}\,\cancel{m}}{s^2}\,\frac{\cancel{m}^2}{\cancel{kg}^2}\left(5.98\,x\,10^{24}\,\cancel{kg}\right)}}$$

$$T = 2\pi\sqrt{\frac{290.12\,x\,10^{18}}{39.89\,x\,10^{13}}s^2} = 2\pi\sqrt{7.273\,x\,10^5\,s^2}$$

$$T = 2\pi\sqrt{72.73\,x\,10^4\,s^2} = 53.57\,x\,10^2\,s = 5357\,s$$

$$T = 5357\,\cancel{s}\left(\frac{1\min}{60\,\cancel{s}}\right) = \boxed{89.3\min}$$

- The moon has a period of 28 days. If the earth's mass is 5.98 x 10^{24} kg, how far is the moon from the earth?

$$T = 28\,\cancel{day}\left(\frac{24\,\cancel{h}}{1\,\cancel{day}}\right)\left(\frac{3600\,s}{1\,\cancel{h}}\right) \qquad T = 2.42\ x10^6\ s$$

$$T = 2\pi\sqrt{\frac{r^3}{Gm}} \qquad \text{square both sides:} \qquad T^2 = 4\pi^2\left(\frac{r^3}{Gm}\right)$$

Solve for r^3:

$$r^3 = T^2 Gm\left(\frac{1}{4\pi^2}\right) \quad r = \sqrt[3]{T^2 Gm\left(\frac{1}{4\pi^2}\right)}$$

$$r = \sqrt[3]{\left(2.42\,10^6\,\cancel{s}\right)^2 6.67\ x\,10^{-11}\frac{\cancel{kg}\ m}{\cancel{s}^2}\frac{m^2}{\cancel{kg}^2}\left(5.98\ x\,10^{24}\,\cancel{kg}\right)\left(\frac{1}{4\pi^2}\right)}$$

$$r = \sqrt[3]{5.923\ x\,10^{25}\,m^3}$$

convert 5.923 x 10^{25} to something times 10^{24} (we chose 10^{24} because 24 is divisible by 3)

$$r = \sqrt[3]{59.23\ x\,10^{24}\,m^3} \quad = \quad \boxed{3.90\ x\,10^8\,m}$$

Gravity in Orbit: We all know that the astronauts in space orbiting the earth are "weightless". Does this mean that there is no gravity in space? Well, no. Most of our spacecraft are in pretty low orbits. The distance between the astronauts and the earth is not that much greater than when they are on the earth. Their weight is only about ten percent less than it is on earth. So why are they weightless?

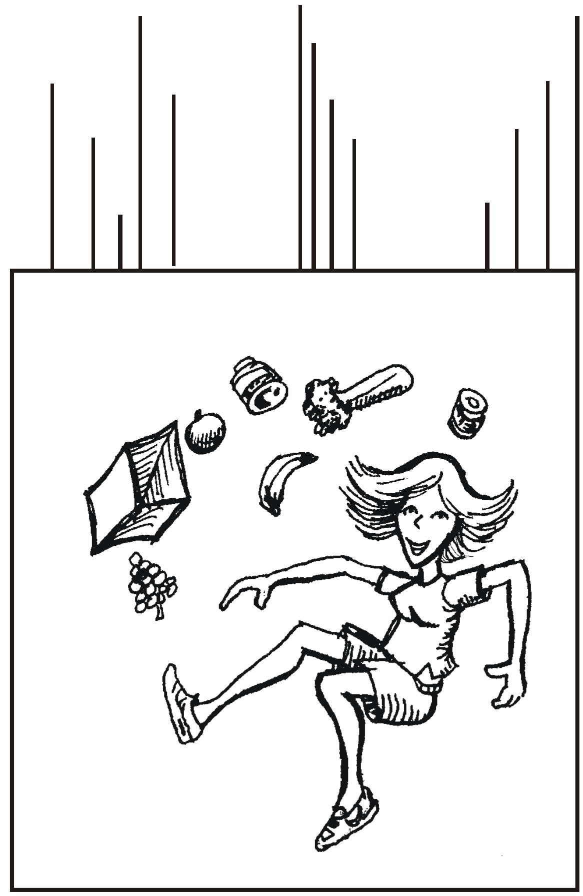
The space shuttle and everything in it are falling towards the earth. It is in a state of freefall. We know that everything falls at the same rate, so everything in the space shuttle is falling at the same speed. Because of this there is no relative motion between the space shuttle and everything in it. There is no sense of up or down and the astronauts no longer feel the force of gravity. It’s like being inside an elevator that is falling down the elevator shaft (the cable broke or something). In a normal elevator, one that isn’t falling, gravity exerts a downward force on everything. If you stand on a bathroom weight scale, you push down on it and it reads out your weight. But now the cable breaks. You are still standing on the scale, but the elevator, the scale, and you are all falling down accelerating at 9.8 meters per second squared. You no longer exert a force on the scale – it is falling at the same speed that you are. It now reads zero.

Any objects in the elevator would appear to be weightless. If you held a ball outward and then released it, it would not appear to fall down (since it is already falling). It would appear to float in space in front of you. You would think, “Hey, cool, there’s no gravity in the elevator. Neat!”

It would be pretty neat too. Until the elevator hits the bottom of the elevator shaft.

Okay, let's transfer this idea to the space shuttle. It is, in effect, a falling elevator, one with the advantage of not crashing into the floor – it never hits the earth!

That's why the astronauts are "weightless".



Free Falling in an elevator